

# The Effects of the Two-Stage PFB Architecture on the Response of the MWA

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In an earlier article I analyzed the operation of the two-stage polyphase filter banks (PFBs) that are used in the MWA-LFD radio telescope and described some of the aspects of how a two-stage filter differs from a single stage filter. Briefly, each coarse channel has some response to signals that are out of the 1.28 MHz channel bandwidth. The response is manifest in neighboring coarse channels but not in the fine channels that are closest to the neighboring coarse channel. Rather, the response tends to appear in the fine channels that differ in frequency from the signal frequency by an integer multiple of 1.28 MHz, i.e., exactly one full coarse channel. In this article, I explore the practical implications of these aspects of the cascaded PFB design.

## Simulation of the PFBs

My simulation of the PFBs captures the algorithms used therein at a high level but does not emulate the process at a minutely detailed level, i.e., the manner in which the calculations are done within the FPGAs. Furthermore, all of the simulation calculations are done using double precision numbers; no attempt has yet been made in this particular simulation code to model the use of small numbers of bits to represent the output from either PFB. Each cycle of operation of each of the PFBs is modelled by an FFT of a set of stacked intervals, or taps, that contain the time bin by time bin products of the input data with a window function.

The window functions that are used in the simulation code are believed to be those used in the actual 32-T PFBs. If not, they must be very close to those actually used. Comparison of laboratory measurements of the instrumental responses to single-frequency sine wave input signals confirms that the simulation is a good model of the actual 32-T system performance. The simulation results shown in Figure 1 were obtained by calculating the response to each of 250 sine waves with frequencies spaced every 20 kHz starting at 252.16 MHz, the central frequency of coarse channel 197. The solid line in the figure shows the responses to individual sine waves. The dashed curves show the results of laboratory measurements made at the Raman Research Institute (RRI) in March of 2011<sup>1</sup>.

One may also compare a frequency spectrum obtained from observations of the sky by the 32-T instrument with simulated results. In this case, the response is to a continuous spectrum of frequencies rather than to one single-frequency signal. To obtain appropriate results, the simulation code was used to calculate the response to each of 2001 sine waves with frequencies spaced every 1 kHz and covering the range of  $\pm 1$  MHz with respect to the center of coarse channel 197. The total power in each of the 128 10 kHz-wide fine channels of coarse channel 197 was formed by summing the power produced in each channel by all

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<sup>1</sup>Prabu performed the measurements and provided the data in the file “Measured\_response\_of\_the\_Coarse\_PFB.14-March-2011.dat” in an email dated 2011 March 14.

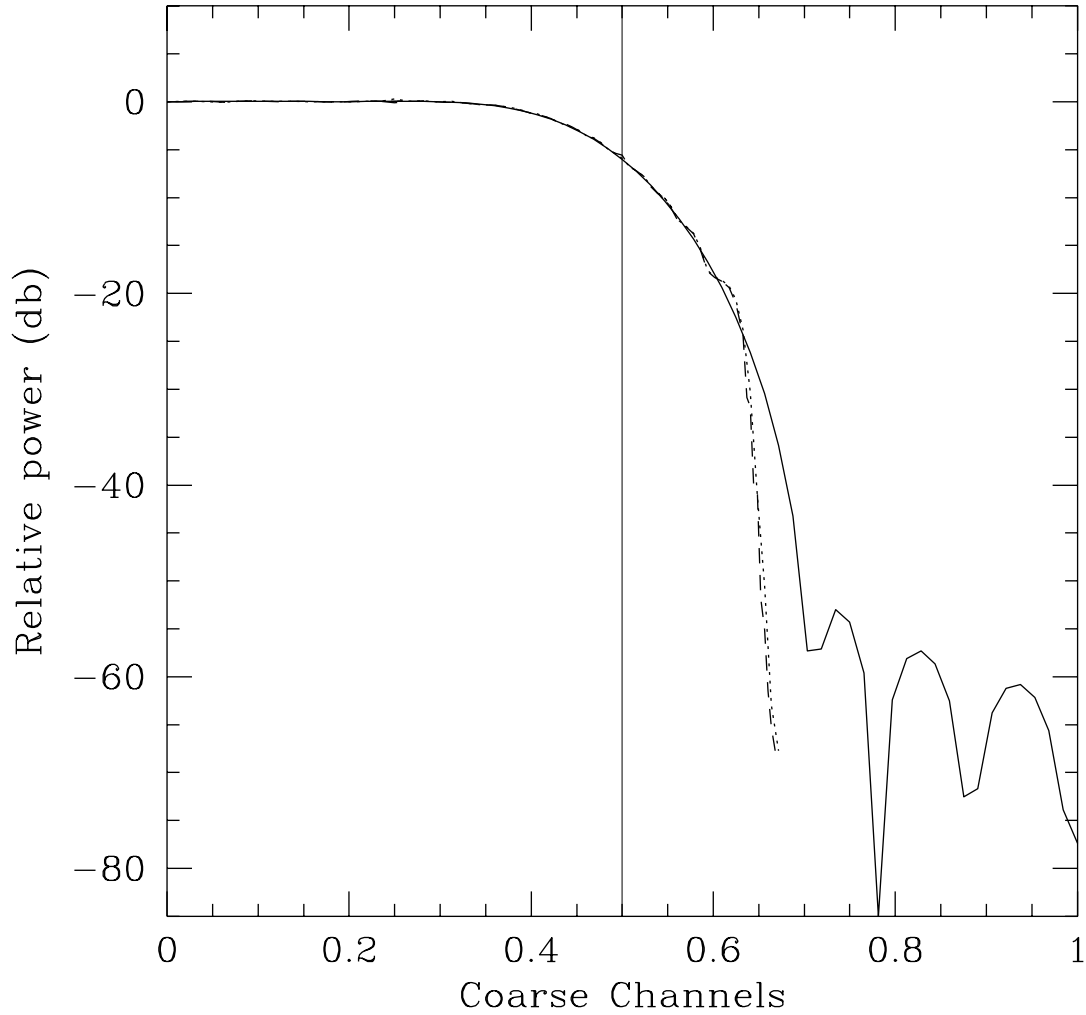


Figure 1: (solid line) The total power in a particular coarse channel from a sine wave input with frequency indicated by the difference in units of coarse channels (1 coarse channel = 1.28 MHz) from the nominal channel center frequency obtained from computer simulations. (short-dashed line) Laboratory measurements made at the Raman Research Institute of the power in a coarse channel in response to sine wave input signals. (long-dashed line) Additional RRI laboratory measurements of the power in a coarse channel in response to sine wave input signals. The results are shown at frequencies reflected around the center of the coarse channel.

of the input signal frequencies. The amplitudes of the input signals were all equal, so that this simulates a flat continuous input spectrum. The results are shown in Figure 2. The top panel of the figure shows the total power summed over all input signal frequencies that end up in each of the 128 fine channels of the particular coarse channel. Ideally, each fine channel would respond only to signals within a specific 10 kHz band. While all of the fine channels have some small response to frequencies just outside their nominal 10 kHz wide bands, the lowest and highest frequency fine channels have substantial responses to signals that differ by close to 1.28 MHz from their nominal central frequencies. The lowest channels respond to signals whose frequencies are just above the upper edge of the 1.28 MHz wide coarse channel, and the highest channels respond to signals whose frequencies are just below the lower bound of the coarse channel. The power-weighted average frequencies of the signals detected in each fine channel are shown in the middle panel, and the power-weighted rms frequency widths are plotted in the bottom panel. The rms widths of the mid-range fine channels are all about 2.7 kHz as expected for channels with 10 kHz wide rectangular bandpasses. Although these results were computed for a specific coarse channel, theory informs us that they must be more or less independent of the coarse channel number and central frequency, and may be compared with the response of a different coarse channel.

The simulation results show that a substantial number of fine channels have comparable responses to two narrow frequency bands that are separated by  $\sim 1.28$  MHz. These fine channels are problematic for use in analysis and imaging. At this time the only recourse is to flag them, i.e., approximately one third of the channels, so that they are not used for imaging or power spectrum purposes.

### **Comparison of the Expected Response with Measurements**

The power going into each fine channel during an observation of the sky is easily obtained from the autocorrelation values. Actually, in the 32-T and 128-T implementations, the 128 fine channels for each coarse channel are rebinned into 32 40 kHz channels. In comparing observations to the simulation results, the term “fine channel” will refer to a rebinned 40-kHz wide channel. Figure 3 shows one such spectrum. A blowup of the region around the first coarse channel is shown in Figure 4 along with a histogram that shows the response from the simulation code, as shown in the top panel of Figure 2, adjusted solely to match the overall power level. The calculated response is quite similar to the observed response except that the drop in response at low and high channels is exaggerated in the simulation results.

This suggests that there is an additional source of white (frequency independent) noise that affects the output values, perhaps stemming from truncation errors due to the limited number of bits in the representation of PFB output values. This possibility was explored by adding a base power to the powers in each of the 32 40 kHz channels. A base equal to 0.14 times the power in central frequency channel (40 kHz channel no. 16 to be precise) was found to yield a good fit to the data (Figure 5).

### **Simulation of Oversampling**

At present it appears that about one-third of the fine frequency channels will need to be flagged. This represents a loss of  $\sim 30\%$  of throughput capability, and so it is of interest to understand what measures could potentially be used to increase the throughput of useful data. One such possibility among the large range of possibilities is oversampling by running the coarse PFB at a higher rate than in the current implementation. I modified my simulation

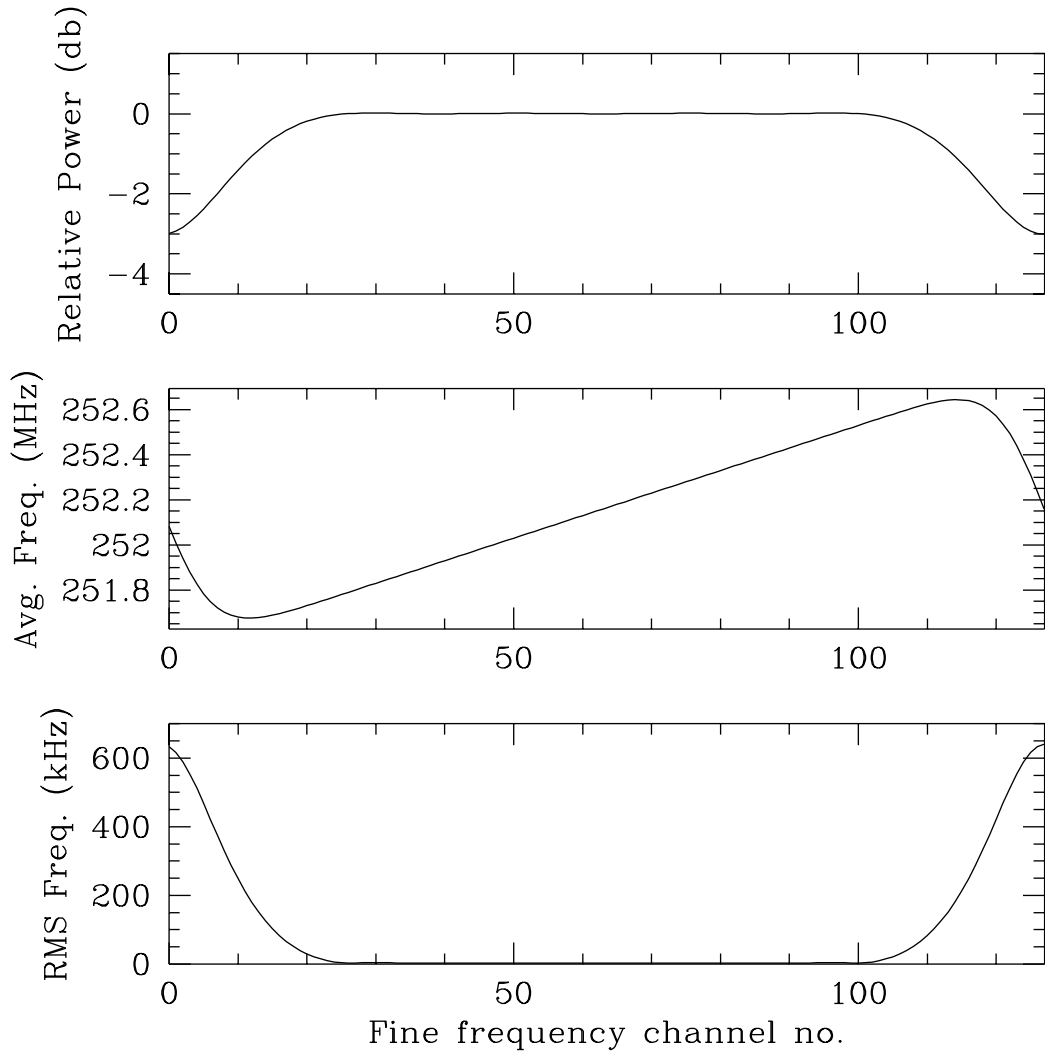


Figure 2: Simulation results for coarse PFB channel no. 197 as a function of fine channel number. (top) Total power in each fine channel received from all simulated signals. (middle) Power-weighted average frequency. (bottom) Power-weighted RMS frequency of the signals going into each fine channel. The RMS frequency of the “plateau” in the middle channels is approximately 2.7 kHz.

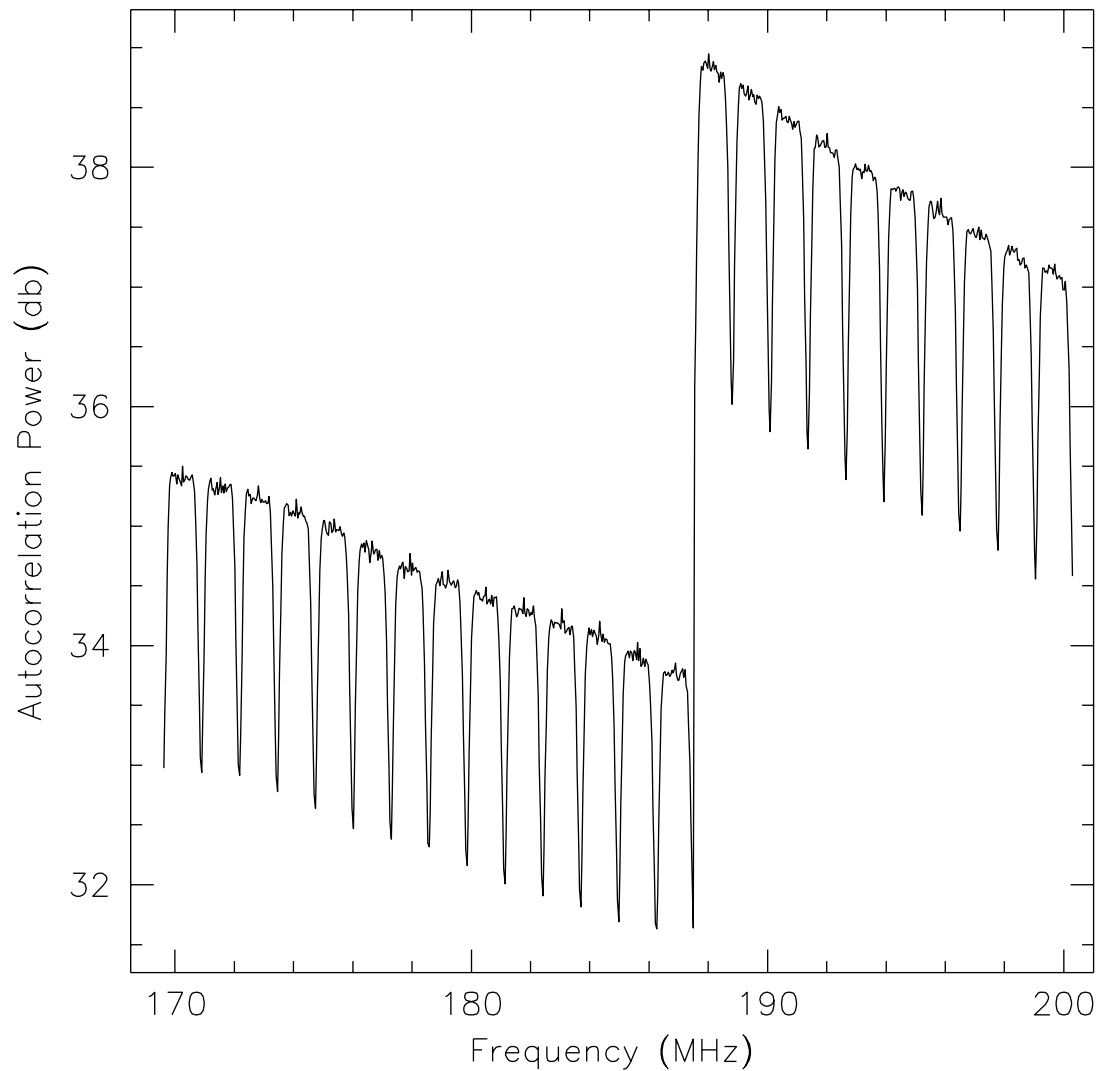


Figure 3: Observed spectrum obtained from the autocorrelations for tile 13 (YY polarization) obtained in a 0.5-s observation in the fall of 2011 (X16). The frequency band comprises 768 40-kHz fine channels in coarse channels 133 through 156. The power gain for coarse channels 147-156 was enhanced through selection of bit fields in the PFB outputs during the observing run.

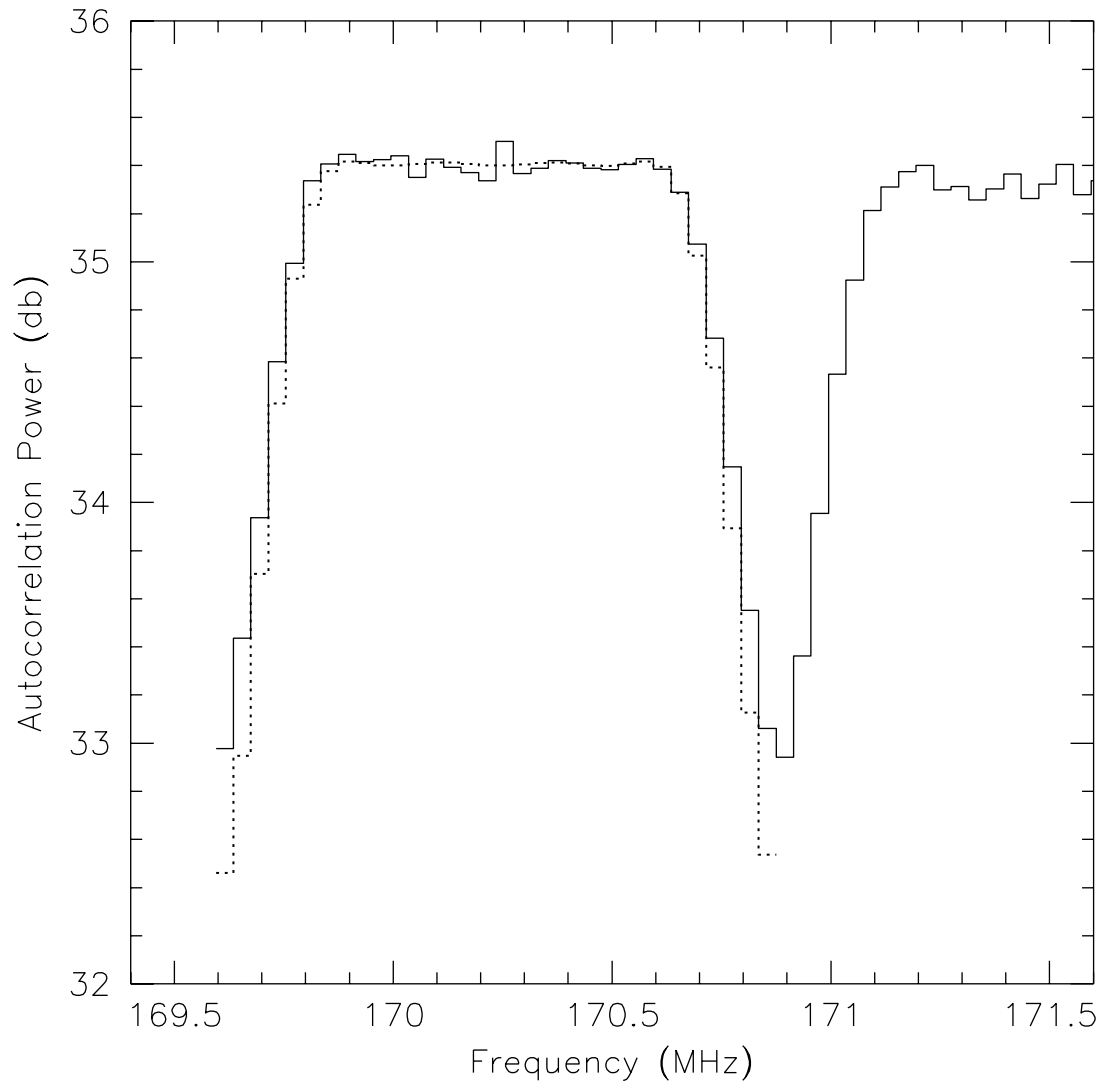


Figure 4: Blowup of the region around the first coarse channel of the autocorrelation power spectrum shown in Figure 3. The dashed histogram shows the response curve obtained from the simulation results without any adjustments except to set the overall power level to match the data.

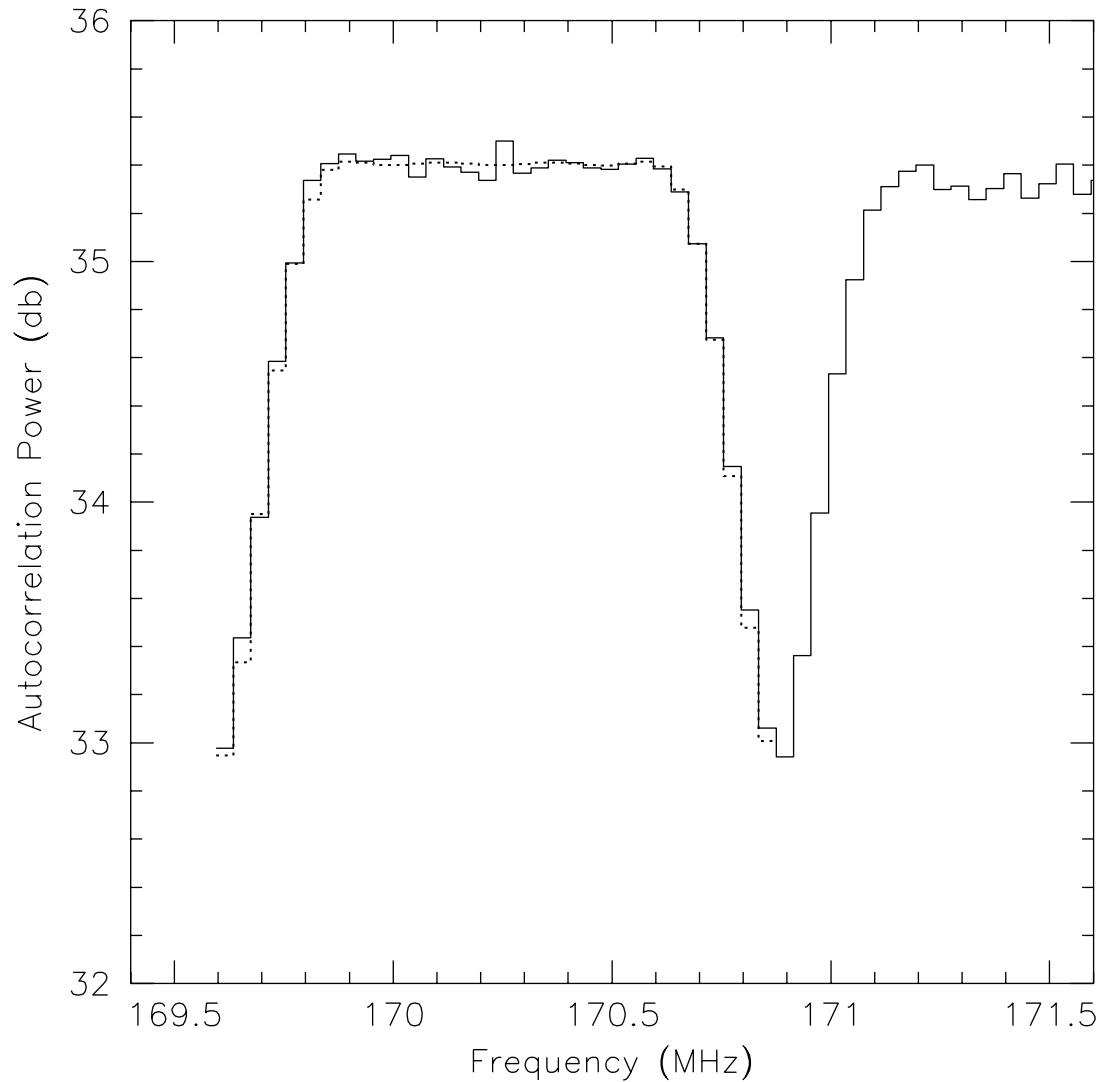


Figure 5: Blowup of the region around the first coarse channel of the autocorrelation power spectrum shown in Figure 3. The dashed histogram shows the response curve obtained from the simulation results with two adjustments. The first one set the overall power level to match the data. The second adjustment was the addition of a white noise, i.e., frequency-independent, component to the powers in each fine channel (see text).

code to allow for this and ran the 8/7 oversampling rate case where the coarse PFB does an FFT once every 448 input samples as opposed to the nominal rate of once every 512 input samples. The A/D conversion rate is assumed to run at the same rate in the critical sampling and oversampling cases. This oversampling version of the simulation was used to explore the response to a continuous input spectrum in a manner identical to that described above (2001 frequencies spaced by 1 kHz and fully covering the region of significant response of a particular coarse channel) for the critical sampling case. The results are shown in Figure 6. In this simulation, the full frequency range of the fine channels is 8/7 times 1.28 MHz, and the central 112 fine channels cover the 1.28 MHz band of the coarse channel. These 112 channels are indicated by vertical bars in the figure. A smaller fraction of the 1.28 MHz-wide coarse channel is affected by aliasing, but the amplitudes of the output values in the lower and higher frequency fine channels are more strongly depressed than in the critical sampling case and are likely to be particularly affected by truncation or rounding noise.

### Modification of Coarse PFB Filter Function

A second possible way to reduce the number of channels that need to be flagged is the modification of the filter function used in the coarse PFB. If this method is effective, it is much simpler to implement than oversampling. I have carried out one useful test of such a possibility by widening the filter function by a factor of 1.25<sup>2</sup> relative to the nominal case. The results of a simulation using this filter function are shown in Figure 7. The core of the coarse PFB frequency response is narrower and falls faster near the coarse channel boundaries. This reduces the number of fine channels that are seriously affected by aliasing. The tradeoff is that the response to sky signals falls off rapidly in the fine channels close to the coarse channel boundaries and are likely to be highly susceptible to the effects of truncation or roundoff noise. Nevertheless, this approach promises to be a practical method of improving the throughput of useful data.

If the coarse PFB were to run with just one tap and without the application of a filter function, then the effects of aliasing are rather severe, as shown in Figure 8. This shows the value of the use of multiple taps and a filter function to the operation of the coarse PFB.

### Discussion

Simulation and analysis of the operation of the MWA PFBs confirms that the fine channels near the lower and upper bounds of each coarse channel are strongly affected by aliasing from signals that are 1.28 MHz outside of the desired fine channel bands. The aliasing is fundamental to the system concept and is not easily remedied; it is not caused by a coding error or errors. This result is consistent with anecdotal reports that these fine channels do not yield good images and should be flagged. It appears that approximately 1/3 of the fine channels are seriously affected. The best case may be that the aliased signals represent contamination that appears as noise in the images. However, it is also possible that the aliased signals may result in the appearance of spurious sources. In any event, we do not yet have a quantitative criterion to be used to set the ranges of channels that need to be flagged. The flagging criterion may even depend on the particular scientific purpose of particular analysis efforts, e.g., in regard to the desired dynamic range of the images to be produced.

The results are also consistent with the descriptions of aliasing seen in laboratory mea-

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<sup>2</sup>Strictly speaking, only the  $\sin(t)/t$  part of the filter function was changed. The function that is used also comprises another time-domain factor that was not modified.



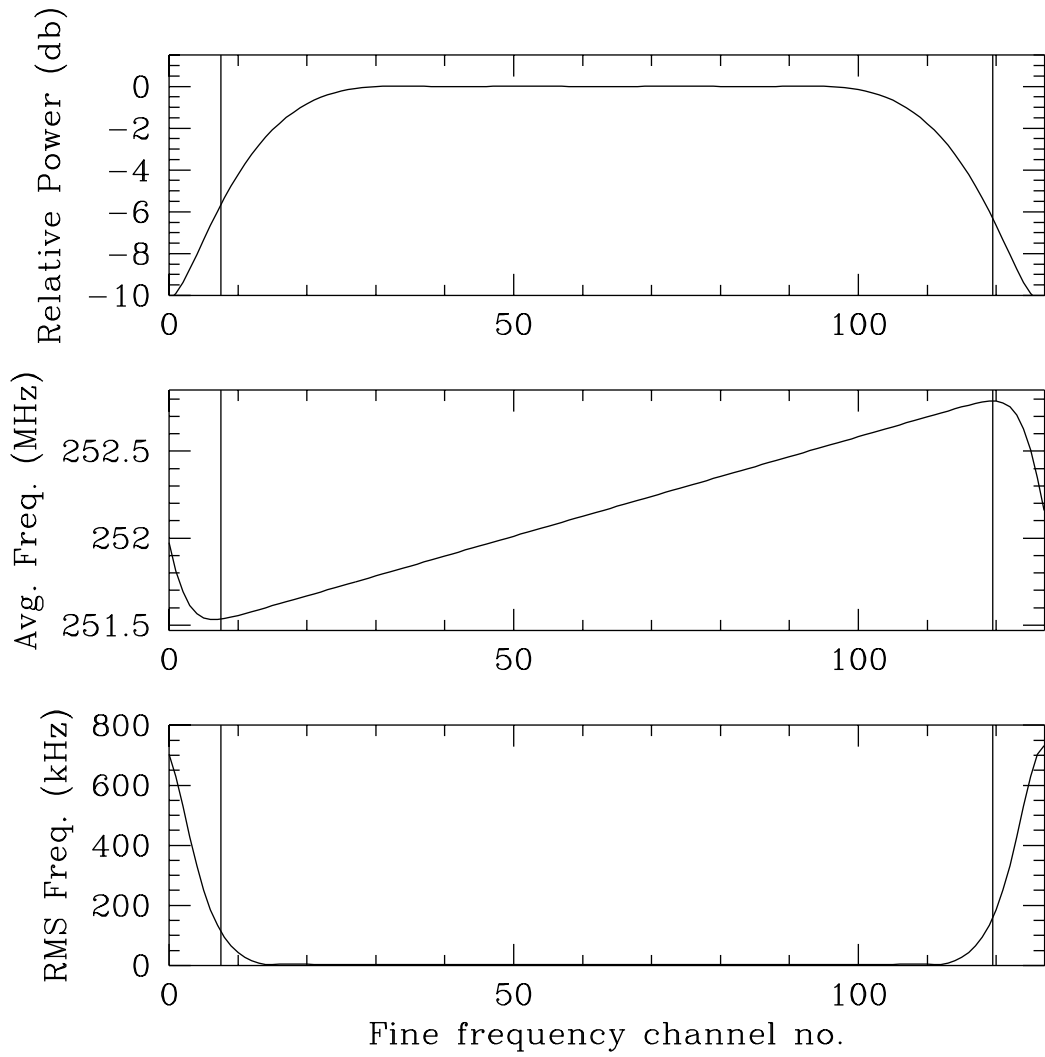


Figure 6: Simulation results for coarse PFB channel no. 197 as a function of fine channel number displayed similarly to the results shown in Fig. 2, but for the case where the coarse PFB runs at  $8/7$  times the baseline rate.

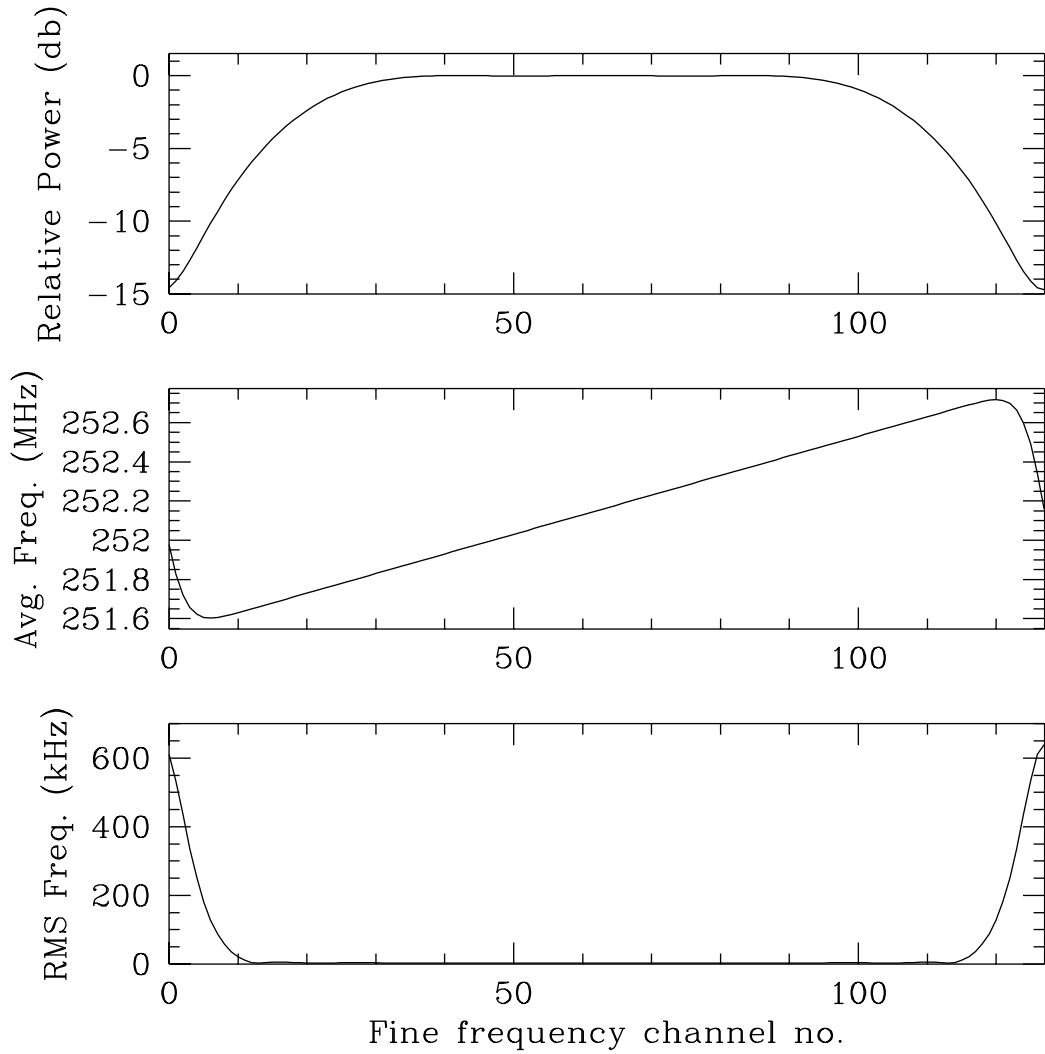


Figure 7: Simulation results for coarse PFB channel no. 197 as a function of fine channel number presented similarly to the results shown in Fig. 2, but for the case where the coarse PFB filter function was widened in the time domain by a factor of 1.25 relative to the nominal case (see text for additional details). As in the baseline case, each fine channel nominally represents a 10 kHz wide band.

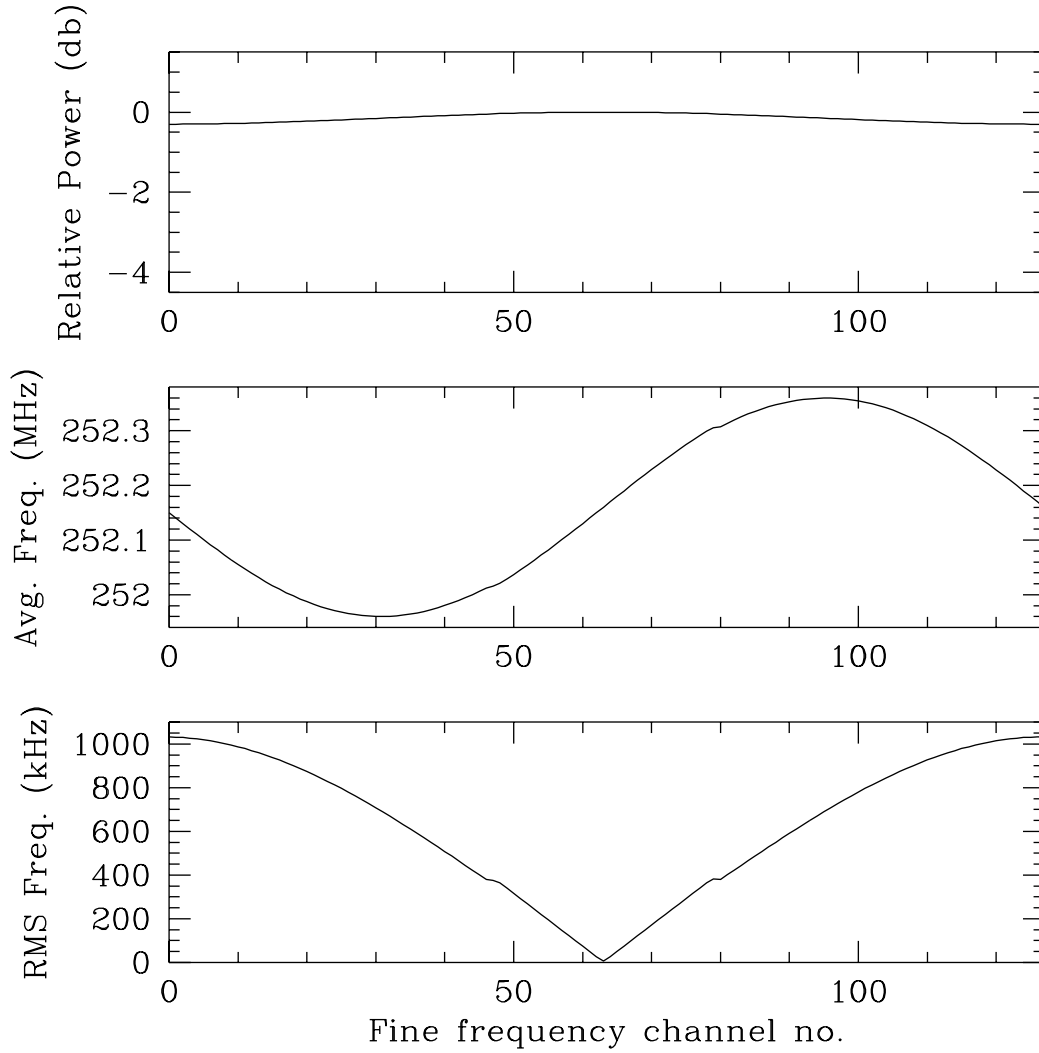


Figure 8: Simulation results for coarse PFB channel no. 197 as a function of fine channel number presented like the results shown in Fig. 2, but for the case where no filter function at all is applied in the coarse PFB and the coarse PFB constitutes one tap only.

surements made at Curtin University in January 2011<sup>3</sup>.

There does not appear to be a simple workaround to eliminate the aliasing problem. The modification of the filter function used in the coarse PFB promises to reduce the severity of the problem and may be the preferred way to address the issue. However, rather little effort has been put to date towards thinking of ideas to guide the development of mitigation options.

The present results could be extended by attempting to incorporate the effects of using small numbers of bits to represent signals within the MWA digital system. Detailed knowledge of the system operation will be required for a useful simulation extension.

### **Recommended Actions**

Below I summarize the topics where I believe that further effort is in order to follow up on the discussion above.

- Provision of a detailed description of the system operation with respect to the bit fields and numbers of bits used for the representation of values including how the output autocorrelations and cross correlations are scaled prior to archiving (how the values in the L-files are related to signal levels in the PFBs)
- Further analysis of the effects of truncation and rounding based on a detailed description of the system operation with respect to the bit fields and numbers of bits used.
- Identification of X16 data obtained at the same time the 2pip system was archiving the output from the coarse PFB (making the 2pip data available for analysis).
- Development of a criterion to scientifically set the ranges of channels that should be flagged.
- Discussion of down-the-road mitigation measures or system architecture changes.

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<sup>3</sup>Reported in “LabTests\_at\_ICRAR\_Curtin-7-15Jan2011.pdf”.